

Stock Investment Portfolio Analysis with Single Index Model

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ABSTRACT

In order to evaluate an optimal portfolio, an important step that investors or investment managers is portfolio analysis. In stock portfolio analysis, methods that can be used include the Markowitz approach and the Single Index Model. This study aims to apply the Single Index Model in finding the beta value of an efficient portfolio line, so that investors can determine the stocks and the proportion of funds needed to form an optimal portfolio. In this study, the data sources used were 1) market share price index that represents market factor or market data, 2) SBI interest rates that represents risk free (r_f) and 3) The share prices of PT Ace Hardware Indonesia Tbk, PT Indocement Tunggal Perkasa Tbk and PT Matahari Putra Prima Tbk. The weight of each share in the active portfolio (W_i^0) at Active Pf A 1.0000 is ACES of 0.1729, INTP of 0.0460 and MPPA of 0.7811. Then the alpha of the ACES active portfolio is 0.0051, INTP is 0.0002 and the MPPA is 0.0184. Then the calculation results show the residual variance in the active ACES portfolio is 0.0041, INTP is 0.0001 and MPPA is 0.0147. The variance of the Optimal Risky Portfolio of the variance index portfolio and the residual variance of the active portfolio is 0.1054.

Keyword: Investment Portfolio, Single Index Model

JEL Clasification: G10, G11, G17

INTRODUCTION

Investment decisions must have a relevant basis in order to achieve the goal of maximizing profits and minimizing risk, where this investment decision can be made by two parties, namely investors or investment managers. Investors or investment managers who invest in shares in the capital market are important to consider several factors, including the amount of capital to be invested, the investment period, the level of risk that will arise, and the amount of return that will be obtained. The level of risk factor and the factor of the amount of return (return) are the main factors that form the basis of making investment decisions, where one of the steps to achieve the objectives of these two factors is to have an investment

portfolio as a diversification step in minimizing investment risk.

In order to form an optimal portfolio, an important step that investors or investment managers must take is portfolio analysis. In stock portfolio analysis, methods that can be used include the Markowitz approach and the Single Index Model.

The Markowitz portfolio model is a portfolio optimization method introduced by Hary Markowitz in the article Portfolio Selection in the Journal of Finance in 1952. Markowitz model states that portfolios can be formed in two ways, namely minimizing variance or maximizing expected return. The Markowitz procedure has several weaknesses, firstly this model requires a very large number of estimates to fill the covariance matrix, these two models cannot provide direction for forecasting the risk premium of

securities which is fundamental to forming an efficient frontier of risky assets.

In 1963, William Sharpe developed the Single Index Model which is a simplification of the Markowitz Model, the Single Index Model provides an easier variance analysis solution when compared to the Markowitz Model analysis which requires using Lagrange Multiplier analysis. The Single Index Model can also be used to calculate expected return and portfolio risk. The Single Index Model can be an alternative in forming an optimal portfolio that is easier for investors or investment managers. By using the Single Index Model approach, we can determine the efficient set of portfolios more simply because the Single Index Model simplifies the amount and type of input (data), as well as the analysis procedure to determine the optimal portfolio. The Single Index Model assumes that the correlation of the returns for each stock occurs because of the security's response to changes in a particular index.

The step that needs to be taken by using the Single Index Model is to find the beta value of the stocks that will be included in the portfolio, in finding the beta value, we can use an assumption / judgment or can use historical beta to calculate past beta which is used as an estimate beta in the future. Historical beta provides important information about future beta. This study aims to apply the Single Index Model in finding the beta value of an efficient portfolio line, so that investors can determine the stocks and the proportion of funds needed to form an optimal portfolio.

LITERATURE REVIEW

Return and Investment Risk

According to Brigham et al (1999), stated that Return is: "measure the financial performance of an investment." return is used in an investment to measure the financial results of a company / an investment. According to (Jogiyanto, 2008) returns can be divided into:

1) Return Realization

Is a return that has occurred. Return is calculated based on historical data. Realized return is important because it is used as a measure of company performance. This historical return is also useful as a basis for determining the expected return and risk in the future. The calculation of realized return here uses total return. Total return is the total of an investment in a certain period.

2) Return Expected

Is a return that is used for making investment decisions. This return is important compared to historical returns because the expected return is the expected return on the investment made. Expected return can be calculated using the expectation value method, which is to multiply each future outcome by its probability of occurrence and add up all the products of this product.

Understanding risk according to Keown (1999), risk is the possibilities that a return will be different from the expected rate of return. According to Jones (2002), there are two types of risk, namely:

a. Systematic risk

Is a risk related to conditions that occur on the market in general, namely interest rate risk, political risk, inflation risk, exchange rate risk and market risk. Also called the risk of not diversification.

b. Non-systematic risk

Is the risk associated with the condition of the company that occurs individually, namely business risk, leverage risk and liquidity risk. Also called diversification risk, residual risk, unique risk, or company-specific risk. So, it can be concluded that risk is the possibility of a real deviation of the rate of return against the expected rate of return. The amount of the risk value can be found by calculating the standard deviation, or by calculating the variance.

Portfolio

Investment portfolios, especially securities investment portfolios, are formed from various combinations of risky assets or a combination of risky assets with non-risk assets in the capital market. The combination of these assets can reach an unlimited number, therefore there is a wide selection of possible portfolios that investors can choose from. With rational assumptions, investors will choose the optimal portfolio. "The optimal portfolio can be determined using the Markowitz model or a single index model. To determine the optimal portfolio with these models, an efficient portfolio is first needed" (Jogiyanto, 2008).

A portfolio can be concluded, namely the investment of various stocks which aims to make an efficient combination of investing in these stocks so that investors can get high returns and can reduce the risk of these investments.

Single Index Model

In 1963, William Sharpe developed a portfolio analysis model called the Single Index Model which simplifies the calculation of the Markowitz model by providing the input parameters required in the calculation of the Markowitz model. The Single Index Model can also be used to calculate expected return and portfolio risk.

"The single index model is based on the observation that the price of a security fluctuates in the direction of the market price index" (Jogiyanto, 2010: 339). In general, the observed stocks that most stocks experience an increase in shares if the stock price index rises, and vice versa if the stock price falls, most stocks experience a decrease in price. This illustrates that the returns of securities may be correlated due to a common reaction (common response) to changes in market value. The single index model can be formulated as follows:

$$R_i = \alpha_i + \beta_i R_m + e_i \quad (1)$$

Where:

R_i = return of the i th security

α_i = the expected value of the security's return that is independent of market returns

β_i = beta which is a coefficient that measures the change in R_i as a result of change in R_m

R_m = rate of return of the market index, also a random variable

e_i = residual error which is a random variable with the same expected value with zero or $E(e_i) = 0$

DATA AND METHODOLOGY

Data

In this study, the data sources used were:

- 1) Market share price index / JCI taken from the datastream. JCI data represents market factor or market data.
- 2) SBI interest rates taken from <http://www.bi.go.id> SBI data represents risk free (rf)
- 3) The share prices of three companies were taken from <http://finance.yahoo.com>

The selected company stock data are three companies, namely:

- a) PT Ace Hardware Indonesia Tbk (ACES),
- b) PT Indocement Tunggal Perkasa Tbk (IMTP),
- c) PT Matahari Putra Prima Tbk (MPPA).

The company data mentioned above were selected with consideration by the level α (Intercept) > 0, where the excess of the portfolio return when the excess of the market return is zero. this selection was made to avoid bias. The data period is monthly data from the prices of the three shares of this company and also monthly data from the IHSB for 5 years (2010 to 2014), so we get 60 data series. The monthly period is taken because the data from SBI (risk free) are issued by Bank Indonesia on a monthly basis.

Methodology

Single Index Model Calculation.

The steps in calculating the single Index Model are as follows:

- A. Calculate the monthly return of each company stock index and IHSB.

Return of stock i is:

$$R_i = \frac{\text{StockReturn}_t - \text{StockReturn}_{t-1}}{\text{StockReturn}_{t-1}} \quad (2)$$

- B. Calculating the Excess Return of each portfolio index by subtracting the risk-free return of each index.

Excess return (t) = return index (t) - risk free (t)
(3)

- C. Compute the average and standard deviation of Excess Returns

- D. To regress the stock excess return (Ri) to the market (Rm)

- E. Perform a portfolio analysis

- a) Calculate the annualized Risk Parameters of the Investable Universe

- b) Correlation of Residuals

- c) Macro Forecast and Forecasts of Alpha Values

- d) Calculate the Optimization Procedure as follows:

1. Calculate annualized Risk Parameters of the Investable Universe

2. Correlation of Residuals

3. Macro Forecast and Forecasts of Alpha Values

4. Calculate Optimizing Procedure as follows:

- 1) Calculate the initial position of each security in the active portfolio

$$\omega_i^0 = \frac{\alpha_i}{\sigma^2(e_i)} \quad (4)$$

- 2) Scale the initial positions so that the portfolio weights to sum to 1 using

$$\omega_i = \frac{\omega_i^0}{\sum_{i=1}^n \omega_i^0} \quad (5)$$

- 3) Calculate the alpha of the active portfolio

$$\alpha_A = \sum_{i=1}^n \omega_i \alpha_i \quad (6)$$

- 4) Calculate the residual variance of the active portfolio

$$\sigma^2(e_A) = \sum_{i=1}^n \omega_i^2 \alpha_i^2 \quad (7)$$

- 5) Calculate the initial position in the active portfolio

$$\omega_A^0 = \frac{\frac{\alpha_A}{\sigma^2(e_A)}}{\frac{E(R_M)}{\sigma^2 M}} \quad (8)$$

- 6) Calculate the beta of the active portfolio

$$\beta_A = \sum_{i=1}^n \omega_i \beta_i \quad (9)$$

- 7) Adjust the initial position in the active portfolio

$$\omega_A^* = \frac{\omega_A^0}{(1 + (1 - \beta_A)\omega_A^0)} \quad (10)$$

- 8) The current weights of the optimal risky portfolio

$$\omega_M^* = 1 - \omega_A^* ; \omega_i^* = \omega_A^* \omega_i \quad (11)$$

- 9) Calculate the risk premium of the optimal risky portfolio from the risk premium of the index portfolio and the alpha of the active portfolio

$$E(R_p) = (\omega_M^* + \omega_A^* \beta_A) E(R_M) + \omega_A^* \alpha_A \quad (12)$$

- 10) Calculate the variance of the optimal risky portfolio from the variance of the index portfolio and the residual variance of the active portfolio

$$\sigma_p^2 = (\omega_M^* + \omega_A^* \beta_A)^2 + [\omega_A^* \sigma(e_A)]^2 \quad (13)$$

- 11) Calculate sharpe ratio from portofolio.

RESULT

Compute the average and standard deviation of Excess Returns

Here the result computation of standard deviation of market index (IHSB), ACES, INTP and MPPA. Form the result we can see

that all stock Standard deviation of excess return are above the market index Standard deviation of excess return. MPPA is the

highest value and INTP is the lowest value among all stock and index.

Table 1. *Standard Deviation of Excess Return*

Stock	SD of excess return
IHSG	0.1558
ACES	0.3868
INTP	0.2824
MPPA	0.4738

Perform a regression of stock excess return (R_i) against the market (R_m), so that the following results are obtained:

Table 2. ACES Regression Statistics

Multiple R	0,29123	
R Square	0,084815	
Adjusted R Square	0,068759	
Standard Error	0.10774083	
Observations	59	
	Coefficients	Std. Error
Intercept	0,029719	0,014202
IHSG	0,722756	0,314465

Table 3. INTP Regression Statistics

Multiple R	0,480847042	
R Square	0,231213878	
Adjusted R Square	0,217726402	
Standard Error	0.072090384	
Observations	59	
	Coefficients	Std. Error
Intercept	0,00353935	0,009503
IHSG	0,87118649	0,210412

Table 4. MPPA Regression Statistics

Multiple R	0,32737389	
R Square	0,107173664	
Adjusted R Square	0,091510044	
Standard Error	0.130352338	
Observations	59	
	Coefficients	Std. Error
Intercept	0,023585759	0,017182878
IHSG	0,99519683	0,380461761

Table 5. Risk Parameters of the Investable Universe (annualized)

	SD of excess return	Beta	SD of Systematic component	SD of Residual	Correlation with IHSG
IHSG	0.1558	1.00	0.1558	0	1
ACES	0.3868	0.72	0.1126	0.3700	0.29
INTP	0.2824	0.87	0.1358	0.2476	0.48
MPPA	0.4738	1.00	0.1551	0.4476	0.33

Table 6. Correlation of Residuals

	ACES	INTP	MPPA
ACES	1	0.004	-0.04
INTP	0.004	1	-0.13
MPPA	-0.04	-0.13	1

Table 7. The Index Model Covariance Matrix

	IHSG	ACES	INTP	MPPA	
Beta	1.00	0.72	0.87	1.00	
JKSE	1.00	0.0243	0.0176	0.0212	0.0242
ACES	0.72	0.0176	0.1496	0.0153	0.0175
INTP	0.87	0.0212	0.0153	0.0797	0.0211
MPPA	1.00	0.0242	0.0175	0.0211	0.2244

To minimize firm specific risk, a portfolio must consist of positive covariances, from

table 7 we can see that all covariances have no negative value.

Table 8. Macro Forecast and Forecasts of Alpha Values

	IHSG	ACES	INTP	MPPA
Alpha	0	0.0297	0.0035	0.0236
Risk premium	0.10	0.1020	0.0907	0.1231

Table 8 shows that INTP risk premium is below the market index but ACES and MPPA are higher risk premium than market index.

All stock forecast of alpha values are higher than market index alpha values.

CONCLUSION

Table 9. *Computation of the Optimal Risky Portfolio*

	IHSG	Active Pf A	ACES	INTP	MPPA	Overall Pf
$\sigma^2(e)$			0.1369	0.0613	0.0241	
$a/\sigma^2(e)$		1.2554	0.2171	0.0577	0.9805	
$W^0(i)$		1.0000	0.1729	0.0460	0.7811	
$[W^0(i)]^2$			0.0299	0.0021	0.6101	
a_A		0.0237	0.0051	0.0002	0.0184	
$\sigma^2(e_A)$		0.0189	0.0041	0.0001	0.0147	
W_0		0.3049				
W^*	0.7004	0.2996	0.0518	0.0138	0.2340	
Beta	1	0.9424	0.1250	0.0401	0.7773	0.9827
Risk premium	0.1000	0.1180	0.0176	0.0042	0.0962	0.1054
SD	0.1558	0.2012				0.1586
Sharpe Ratio	0.6417	0.5864				0.6645

The weight of each share in the active portfolio (W_i) at Active Pf A 1.0000 is ACES of 0.1729, INTP of 0.0460 and MPPA of 0.7811. Then the alpha of the ACES active portfolio is 0.0051, INTP is 0.0002 and the MPPA is 0.0184. Then the calculation results show the residual variance in the active ACES portfolio is 0.0041, INTP is 0.0001 and MPPA is 0.0147. The variance of the Optimal Risky Portfolio of the variance index portfolio and the residual variance of the active portfolio is 0.1054. The complete results of the asset optimization process for the three securities are shown in table 9.

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